Algebra 3

Compulsory exercises, first set
Første sæt obligatoriske opgaver

9. February 2010

The students in the course have to submit a written solution to the following exercises in the mailbox of the instructor (Martin Wedel Jacobsen) on tuesday 16. February at the latest at the start of the exercise classes. The written solutions (in Danish or English) should be carefully written with references to quoted results from the notes and the exercises.

Exercise C1.1: List three groups of order 66 which are not isomorphic. Your answer should include a proof for the fact that they are not isomorphic.

Hint: You may use the fact that two finite isomorphic groups must have the same number of elements of any given order or that their center must have the same order. Proofs of these facts are not required.

Exercise C1.2: Let G be the subgroup of the symmetric group S_{12} generated by the permutations

$\alpha = (1,2,3,4,5,6)$ and $\beta = (1,7)(2,8)(3,9)(4,10)(5,11)(6,12)$.

(1) Show that G contains the permutations

$(1,2,3,4,5,6)(7,8,9,10,11,12)$ and $(1,2,3,4,5,6)(7,9,11)(8,10,12)$.

(2) Show that one of the permutations from (1) commutes with with α and β. Explain why this element is contained in the center $Z(G)$ of G.

Exercise C1.3: Let $\phi : G \to H$ be a homomorphism from the group G to the group H.

(1) Show ϕ maps a commutator in G onto a commutator in H.

(2) If $G^{(1)}$ and $H^{(1)}$ are the commutator subgroups of G and H repectively show that $\phi(G^{(1)}) \subseteq H^{(1)}$.

Suppose now in the rest of this exercise that ϕ is an *epimorphism*.

(3) Show that $\phi(G^{(1)}) = H^{(1)}$.

(4) Assume that G is a finite group. Show that $|H : H^{(1)}|$ divides $|G : G^{(1)}|$.

Hint: Let $\psi = \kappa \circ \phi$, where $\kappa : H \to H/H^{(1)}$ is the canonical epimorphism. Show that $G^{(1)} \subseteq \mathrm{Ker}(\psi)$ and that ψ is an epimorphism. Apply the Homomorphism theorem.