

LaTeX Course 2006-7

Final Exam Project

Analysis of a Function

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Formalities

This is the final project for the LaTeX course in intermediate week 2, 2006-7. The project has to be written and turned in by each participant individually. It is allowed to get help from lecturer, instructors and fellow students. The front page of the report should clearly state the instructors name and the name and CPR-number for the student.

The report is to be handed in to the instructor no later than

Friday the 10th of November 2006.

If the deadline is not met the project is to be handed in directly to the lecturer. You need a good reason to hand in the project late.

The project can be done either in Danish or English, but it has to be done solely in one language. It is evaluated as pass/fail.

Introduction

Imagine that you are going to do an analysis of a function as described in the problems below. Often you would do that by solving some of the problems by hand and other of the problems using Maple. After that you would write a nice report in $\text{\LaTeX} 2_{\epsilon}$ based on your hand-written notes and the Maple worksheet.

In this project we already made some of the hand-written calculations and solved some of the problems in Maple. We also provided handwritten notes covering the mathematical background for analysis of functions. It is then your job to write the nice report. Not all the questions are answered in the notes and the Maple worksheet, so you will also have to use your mathematical skills.

Problems

We consider the function $f : \mathbb{R} \setminus \{\pm 1\} \rightarrow \mathbb{R}$ given by

$$f(x) = \begin{cases} \cos(2x)/\ln(|x|), & x \in \mathbb{R} \setminus \{0, \pm 1\}, \\ 0, & x = 0. \end{cases}$$

All the following problems must be solved in the report.

- (1) Use Maple to plot the function f on the interval $[-2, 2]$, using the exact command `plot(f(x), x=-2..2)`. Comment on the graph and its special behavior.

- (2) Show that f is continuous in $x = 0$ and therefore in all real numbers except ± 1 .
- (3) Make a detailed plot of f on a small interval around $x = 0$ (for example $[-0.1, 0.1]$). The plot should clearly show that the function is continuous in 0.
- (4) Show that the function f is not differentiable in $x = 0$. Use Maple to find an expression for $f'(x)$ for $x \notin \{0, \pm 1\}$. (Note that $\mathbf{abs}(x, 1)$ is not a well-know mathematical expression, so you will have to find out what it means and how the meaning can be used to simplify the answer further.)
- (5) Use Maple to calculate the mean value of f on the interval used in question (3). Make a plot of both f and the mean on the given interval.

Additional material

The hand-written note with *some* of the calculations needed for solving the problems is attach to the project. The Maple worksheet is found on the homepage. It is emphasized that just copying the hand-written notes is not sufficient to answer the questions!

The necessary mathematical background material is also attached to the project as three hand-written pages. The background material must also be included in the $\text{\LaTeX} 2_{\epsilon}$ report. You can either incorporate the material as a part of the text or you can put in an appendix. Remember to refer to the definitions and theorems when you use them.

The report

The purpose of this project is to write a report in $\text{\LaTeX} 2_{\epsilon}$. The report should be coherent and contain arguments, formulas, calculations, plots etc. to solve the problems posed in the previous section. Some of the Maple commands used to answer the questions should also be included and explained in the report (use `verbatim` to write the commands).

It is important that the report is presented clear and readable. Then main emphasis is on the report, but the mathematical part should still be correct!

It may take a long time to do the project, so start early!

English – Danish word-list for mathematical words

We have provided a small word-list for mathematical word in English you might not be familiarly with.

analysis of a function funktionsundersøgelse

continuous kontinuert

derivative afledt, differentialkvotient

differentiable differentiabel

limit grænseværdi

neighborhood omegn

mean value middelværdi

theorem sætning, teorem

real numbers reelle tal (\mathbb{R})