GEOM2, Fall 2011. TEST EXERCISES

Hand in solutions to Anssi at the exercise class on Friday Jan 6, 2012. The solutions will be graded PASSED or FAILED. A passed test is required for participation in the exam in the end of January. It is not necessary for passing that you have a complete and correct answer to every exercise, but your solution should demonstrate an understanding of the various aspects of the theory which is represented by the different questions.

RULES. You are allowed to collaborate, but your final answers must be worked out individually. You are free to consult other sources as long as you do not copy-paste. If you ask one of the teachers we will not give hints, but we will of course reply if your question is caused by an error in one of the given exercises -in that case a correction will be posted on Absalon, and (if possible) the participants will be notified by email. We will however answer general questions, also when they are indirectly related to the test. For example, if you do not understand something in the lecture notes, then you are welcome to ask.

1 Solve Exercise 16 from Chapter 1.

The solution should of course include proof that $M \setminus \{P, Q\}$ is a manifold in $\mathbb{R}^3$, but in addition you are requested to prove that $M \setminus \{P\}$ is *not* a manifold in $\mathbb{R}^3$.

2 Solve Exercise 12 from Chapter 2.

3 Consider the cylinder and sphere of radius 1

$$M = \{x \in \mathbb{R}^3 \mid x_1^2 + x_2^2 = 1\},$$

$$N = \{x \in \mathbb{R}^3 \mid x_1^2 + x_2^2 + x_3^2 = 1\}.$$

a. Prove that M and N have the same tangent space at each point $p \in M \cap N$.

b. Show that the map $M \to N$ given by $x \mapsto x/\|x\|$ is smooth and determine its differential at $p \in M \cap N$.

4 Solve Exercise 9 from Chapter 4.

5 Solve Exercise 14 from Chapter 4.

6 Let X be a compact topological space, and let $A \subset X$ be a subset with the property that each element $x \in X$ has a neighborhood which contains at most one point from A. Prove that A is finite.

7 Let M be an abstract manifold and assume that there exists a locally finite atlas for M. Let $M = \cup_{\beta \in B} E_\beta$ be an arbitrary open covering. Show that M has a partition of unity $(f_i)_{i \in I}$, such that for each i the support $\operatorname{supp} f_i$ is compact and contained in E_β for some $\beta \in B$.